The Application of Options Volatility Arbitrage Strategy in Cryptocurrency Market

Authors: Gate.io Research, Tlntin Deng, Guin Peng, Jill Chow

Translators: Tessa Hu, Lisa Liu, Page Xu, Patrick Velleman

Abstract

In traditional financial markets, by building different models to simulate the market performance of financial products, many investors attempt to predict the future performance of these products in order to actualize bigger profits. As a new trading market, the cryptocurrency market has some similarities with traditional financial markets, which implies the price of cryptocurrencies could potentially also be predicted by applying different mathematical models. To start with, this report will present an overview of what Options are, including their definition, sources of value, pricing methods, etc., followed by the emphasis of the introduction of volatility features and how arbitrage strategy works. Finally, ARCH and GARCH models both have been built and GARCH has been applied to make simulations and predictions on the volatility of historical BTC price. The results show that a volatility arbitrage strategy can also be applied to cryptocurrency markets.

Key Takeaways

◆ According to our conclusion, the higher the volatility in Options, the more potential value, which means the premium would also be higher. Consequently, Options would be more valuable, if the Options period would be longer, making the range of volatility wider.

◆ Seeing the requirements that need to be met for the pricing model of Black—Scholes (BS) for traditional Options trading, the model can also be applied to cryptocurrency Options trading. Furthermore, compared with traditional financial markets, the cryptocurrency market is more free and open, making the application of the Black—Scholes model more effective with cryptocurrency markets.

◆ Volatility is the only parameter that could not be obtained by direct observation with the Black—Scholes model. The change in volatility can greatly influence the value of Options. That is why the majority of professional Options traders try to predict changes in the Options market with the aid of volatility. Additionally, based on three major features of volatility, which are Serial Correlation, Mean Reversion and Momentum Effect, most traders can decide to buy or sell by predicting the changes in volatility.

◆ Gate.io Research built models exploring a logarithmic rate of return of daily BTC prices, conducted a series of verifications on the autocorrelation of BTC volatility,

and constructed a GARCH model. With these, it is found that comparatively accurate predictions on the trend of BTC ROI can be made, while predictions on the exact increase or decrease of ROI are not as effective, and predictions on the volatility of ROI are only applicable over the next 2 consecutive days.

1 Overview of Options

1.1 Introduction to Options

1.1.1 Definition of Options

Options are a type of contract that was first introduced in the US and European markets in the late 1700s. An Options contract gives holders the rights to buy or sell an amount of asset at a predetermined price at or before a specified date. Different from Futures contracts, Options grant holders rights, but incur no obligations, which means holders are able to choose to exercise the Options or not.

1.1.2 Types of Options

Currently, there are two categories of Options defined by their exercise restrictions: American–style Options and European–style Options. In comparison, American Options are more flexible as they allow holders to exercise the Options contract at any time prior to and including its expiration date. In this regard, American Options are more friendly to Options holders and less friendly to Options sellers. Accordingly, however, investors need to pay a higher premium when they buy American style call Options (the Options buyer expects a price surge) or put Options (the Options buyer expects a price slump).

As there remains no crypto exchange that offers American Options, in this report, we focus our study on European-style Options. For holders with European Options, there will be four scenarios regarding the exercise price (also called strike price) and the actual price of the underlying asset when the expiration date arrives:

	Call Option	Put Option
Price Surge	In the Money (ITM): The Options holder exercises the Option at the exercise price	OTM: The Options holder gives up the Option, and the seller gets the premium
Price Slump	Out of the Money (OTM): The Options holder gives up the Option, and the seller gets the premium	ITM: The Options holder exercises the Option at the exercise price

Note: At the Money (ATM) refers to a situation where the strike price is the same as the actual price.

It is shown in the diagram that Options holders are granted rights but no obligations, regardless of call or put Options. This means holders' risks are limited to the loss of a premium. However, for Options sellers, as they only have obligations, their maximum profit is the premium.

1.2 How Options Are Priced

To make a sensible and reasonable investment, investors have to choose an appropriate financial instrument. In this case, before making a final investment decision, they have to

learn where the value of the financial product comes from. For Options, their value derives from their intrinsic value and time value.

1.2.1 Intrinsic Value of Options

The intrinsic value of Options is determined by the spread between the Options exercise price and the market price of the underlying asset. It lies in the profit when holders buy or sell the underlying asset at an exercise price that is higher than the market price. For Options investors, only ITM Options have intrinsic value, while ATM and OTM Options do not, when the predetermined date strikes.

1.2.2 Time Value

The time value of Options mainly reflects in the risk-free rate of return (interest rate) and the Options volatility.
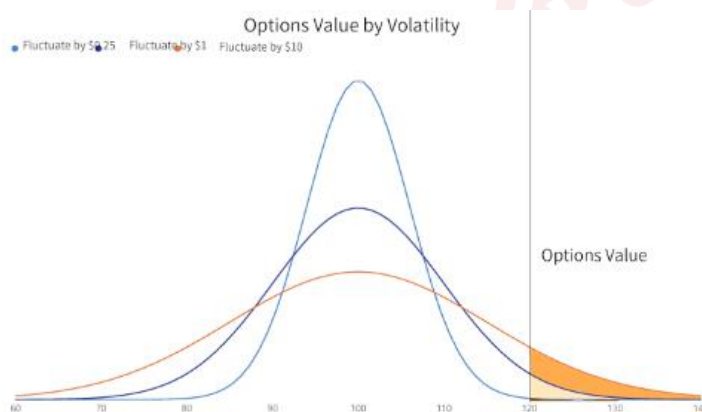
(1) Risk-free Rate of Return

The risk-free rate of return is normally represented by the interest rate of short-term government bonds, which is the most complicated element that affects the valuation of Options. When the risk-free rate increases, the present value of the exercise price decreases; to put it another way, for call Options holders, the present value of the exercise price reduces, making the Options contract more valuable. In this case, the price of call Options will see a surge. While for put Options holders who aim to sell the

underlying asset at the exercise price, the rise in risk–free rate indicates that the opportunity cost will increase, and the Options become less valuable.

(2) Value from Volatility

According to statisticians, if the time of observation is long enough, the distribution of prices of most stocks or indexes would be distributed normally. Based on the theory, if the share price of a financial product is $100 USD, we will have a diagram of the distributions of Options value when the market fluctuates by $0.25, $1, and $10 USD.

The tiny red coverage between the curve in light blue and the X axis shows the Options value when the market fluctuates by $0.25 USD. It has calculated that the Options is $0.05 per   share at this point. When the market fluctuates by $1 USD, the Options value is the red and pale–yellow coverage between the dark blue curve and the X axis, with a share price of $0.75 USD. When the market fluctuates by $10 USD, the Options value is the orange coverage between the orange curve and X axis,

Options Value by Volatility

Note：This diagram does not represent the actual calculation results

with a price of $8 USD per share. The diagram indicates that **for call Options, the higher the volatility is, the stronger the possibility that the actual price of underlying assets would be higher than the exercise price is, and consequently the Options would be more**

valuable. Likewise, for put Options, when the volatility is high, there is a stronger possibility that the actual price of underlying assets would be lower than the exercise price, and consequently the Options would be more valuable. These two situations are mirrors to each other.

1.3 Pricing Options with the BS Model

During Options trading, holders need to pay a certain amount of premium for Options. Based on the conclusion above, Options value is closely related to volatility and time. During the sale of Options, the premium needs to be priced. Currently, the most frequently-used valuation tool is the BS model.

1.3.1   Assumptions for a BS Model

(1) During the lifespan of Options, no dividend is distributed of the underlying asset of Options holders, nor would there be any kind of other allocation.

(2) There is no trading fee for trading the underlying assets or Options

(3) Short-term risk-free rate is known and remains unchanged during the life of the Options

(4) Any security buyer can borrow any amount of funds with a short-term risk-free rate

(5) Short-selling is allowed and short-sellers will immediately acquire the funds from securities sold at the market price on short-selling day;

(6) Call Options can only be exercised on the day of expiration;

(7) All securities are tradable on a 24/7 basis with a random walk price.

While traditional financial markets hardly meet the requirements in (1), (2), and (7), crypto markets perfectly meet all requirements except for (2). That being said, many exchanges provide market makers with huge discounted fees, some even offering negative trading fees. As a result, the BS model is applicable to cryptocurrency Options trading. Moreover, crypto markets are more flexible and open compared with traditional financial markets, making the model work better with crypto markets.

1.3.2   Calculating the Options price with the BS Model

Based on the assumptions, the price of European Options is calculated as follows:

$$C = SN(d_1) - Xe^{-rt}N(d_2)$$

$$P = Xe^{-rt}N(-d_2) - SN(-d_1)$$

Among the formulas,

$$d_1 = \frac{ln\left(S/K\right) + \left(r - \sigma^2/2\right) T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln\left(S/K\right) + \left(r - \sigma^2/2\right) T}{\sigma\sqrt{T}}$$

C — the current value of Call Options

P — the current value of Put Options

X — the exercise price of Options

S — the current price of the underlying asset

t — time prior to the expiration date (year)

r — the continuously compounded annual risk–free interest rate

e — the base of natural logarithms, and its value is appropriately 2.7183

N(d) — the Cumulative Distribution Function for standard normal distribution; it represents the probability where the deviation value is smaller than value 'd,' the degree of volatility. Given that the normal distribution of Options prices varies from one to another, the average values of Options and the standard deviations (or the volatility) are different. The Function could refer to the possibility when Options are exercised with the volatility (σ) of the underlying asset used as a market risk and the price of the underlying assets used to price Options.

From the model, the value of Options closely relates to 5 factors: strike price, spot price, expiration date, risk–free interest rate and the probability of volatility (σ) to be exercised. Generally, the former four factors can be implemented directly during the process of purchasing the Options, whereas the volatility (σ) is not able to be observed intuitively.

2 Principles of Hedging Strategies

As the only unknown factor in the BS model, Options volatility plays an important role in the Options value. In this situation, numerous professional Options traders tend to apply volatility to forecast the movement of the Options market.

2.1 Features of Volatility

To some extent, the variation of volatility can be observed and predicted, the principle of which is similar to the weather report. For example, if we know today's weather and attempt to predict tomorrow's or the day after tomorrow's weather, we can look into the same-period of weather change of last year to predict the weather.

All in all, there are three characteristics of volatility:

(1) Correlation property for sequences. If the provided data is in shortage, it can be deemed that the late volatility is consistent with the current volatility.

(2) Mean reversion. If the provided data is sufficient, it can be considered that the volatility conforms to a rule to some extent. Besides, the late volatility will revert to the historical value.

(3) Momentum effect. If the volatility has already moved towards a certain direction it can be regarded that the late volatility will keep moving forward along with that trend.

Based on the three characteristics of volatility, most traders can make decisions on the trade direction by forecasting the change of volatility.

2.2 Classification of Volatility

Theoretically, the Options value performs better if the volatility is bigger. Nevertheless, it does not mean the higher the volatility is the better the Options are. The expiration date and the amount of premium should be taken into account as well. Investors are supposed to learn more about the classification of volatility such as history volatility, future volatility, forecast volatility and implied volatility before taking actions.

Type of Volatility	Definition
Historical Volatility	Volatility calculated based on the historical prices of the underlying asset
Future Realized Volatility	Volatility calculated based on the future prices of the underlying asset
Forecasted Future Realized Volatility	A prediction on the future realized volatility through particular models
IV	Underlying volatility calculated based on the BS model

2.3 How Volatility Arbitrage Works

As analyzed in section 1.3.2, an IV reveals the entire market's opinion over the potential movements of the underlying asset of an option. Involved in volatility arbitrage, options investors tend to make a forecast of the future volatility of the underlying asset with particular models such as a GARCH model. By doing so, traders are likely to make a profit by taking advantage of the difference between the IV and the forecasted volatility as they look for options where the IV is significantly higher or lower than the prediction.

It's understandable that choosing a right asset and buying the asset at an appropriate time are key to delivering successful investments, which is true for conventional financial instruments and works for financial derivatives such as options, as well. For example, if the IV of an underlying asset is calculated as 15%, while its forecasted future price volatility is computed as 50% through a GARCH model, then the trader who believes the IV is too low and the actual volatility will certainly go higher could purchase a call option. In this case, the investor is said to be long a call option.

During the actual process of volatility arbitrage, options investors tend to adopt a Delta neutral portfolio as an attempt to hedge financial risks and reduce potential losses. Here, Delta is a hedge ratio used to measure the degree to which the option price (premium) is exposed to shifts in the price of the underlying asset.

Here is how the Delta neutral strategy works:

A trader who buys a call option of a derivative sells a corresponding amount of the option's underlying asset simultaneously.

A trader who sells a call option of a derivative buys a corresponding amount of the option's underlying asset simultaneously.

A trader who buys a put option of a derivative buys a corresponding amount of the option's underlying asset simultaneously.

A trader who sells a put option of a derivative sells a corresponding amount of the option's underlying asset simultaneously.

For instance, a trader purchases 20 call options on BTC at a unit price of $1 USD. Assume each BTC option can be swapped for 0.1 BTC and BTC is currently trading at $8,000 USD; if the call option has a Delta of 0.2 (namely, the options price goes up or down by $0.2 USD as the BTC price increases or decreases by $1 USD), then the trader needs to sell 0.4 (0.2*20*0.1) BTC to make a Delta neutral portfolio.

If the value of Delta remains unchanged, earnings from Delta neutral trading would be zero throughout time, if not negative, with handling fees taken into consideration. During the actual trading process, though, the value of Delta shifts as the prices of the option and its underlying fluctuate. In this scenario, when traders implement volatility arbitrage in conjunction with a Delta neutral portfolio, their profits are in close correlation with the discrepancy between the underlying's future price volatility and its IV. Due to the

unavailability of the future realized volatility, options traders often use a prediction of the future volatility to do the job.

3　How to Forecast Volatility with GARCH

3.1 ARCH and GARCH Models

An extension of the Autoregressive Conditional Heteroscedasticity (ARCH) model, the Generalized ARCH (GARCH) model is currently the most widely applied tool for volatility predictions by financial modeling professionals.

3.1.1　ARCH Models

An ARCH model is a time-series statistical model where the error term follows normal distribution with a mean of zero and a variance being conditionally heteroskedastic. Meanwhile, the variance of the error term is affected by variances preceding it, which is referred to as "autoregressive".

The formulation is defined by:

$$y_t = \beta x_t + \varepsilon_t$$

where y_t is the dependent variable, x_t is the independent variable, and ε_t is the error term.

When ε_t^2 fits an Autoregressive (q), or an AR (q) process, then

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 + \eta_t$$

$$\begin{cases} \alpha_0 > 0 \\ \alpha_i \geq 0, \sum_{i=1}^{q} \alpha_i < 1, \quad i = 1, \dots, q \end{cases}$$

Meanwhile , when t is a sequence of independently and identically distributed (IID) variables, with E(t)=0 and the variance being a constant, it indicates that an ARCH(p) process could be applied.

It can be deduced from the above formula that past volatilities would have a positive impact on the future volatility and the extent of the impact is dependent on the i parameter. In other words, under the ARCH framework, large shocks tend to be followed by another large shock, while small shocks appear to be followed by another small shock.

To ensure that the process is stationary and the conditional variance ht is not negative, i≥0 and i=1,,q are imposed. Meanwhile, as the historical volatility in the ARCH model is expressed by t-q2, it suggests positive fluctuations have the same effect as negative shocks on the future volatility, which is in contradiction with the actual situation. In light of the weaknesses of the ARCH model, the GARCH model is more widely used to forecast financial volatilities in the real world.

3.1.2 GARCH Models

The GARCH is commonly known as the generalized ARCH model. When ε_t^2 fits an Autoregressive Moving Average (p, q), or an ARMA (p,q) process, then

$$\varepsilon_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \cdots + \alpha_q\varepsilon_{t-q}^2 + \beta_1\sigma_{t-1}^2 + \cdots + \beta_p\sigma_{t-p}^2 + \eta_t$$

$$\left\{ \begin{array}{l} \alpha_0 > 0 \\ \alpha_i \geq 0, \sum_{i=1}^{q} \alpha_i < 1, i = 1, \ldots, q \\ \beta_j \geq 0, \sum_{j=1}^{p} \beta_j < 1, j = 1, \ldots, p \end{array} \right.$$

Meanwhile, when t is a sequence of independently and identically distributed (IID) random variables, with E(t)=0 and the variance being a constant, then it is appropriate to apply a GARCH(p, q) process; when p=0, the GARCH(p, q) process is equivalent to an ARCH(q) process.

Low-order A GARCH (p, q) models are generally preferred to a high-order ARCH model, for reasons that there are fewer parameters to estimate. The restrictive conditions on the parameters of i and j indicate that the GARCH model is wide-sense stationary.

3.2 Constructing an ARCH Model

Before modeling the future volatility with ARCH and GARCH models, the following test procedures are required:

① Create a mean equation and test the stationarity of the data

② Conduct an ARCH test on the residuals of the mean equation

③ If there exists ARCH effects, then it would be appropriate to proceed with an ARCH model

④ Test the goodness of fit of the ARCH model; adopt a different model if there is a lack of fit

3.2.1 Stationarity Test

The data sample for this modeling is collected from the daily BTC price from January 1, 2017 to May 27, 2020. The log return is defined as $r_t = \ln(P_t) - \ln(P_{t-1})$, where P_t refers to the price of BTC at day t.

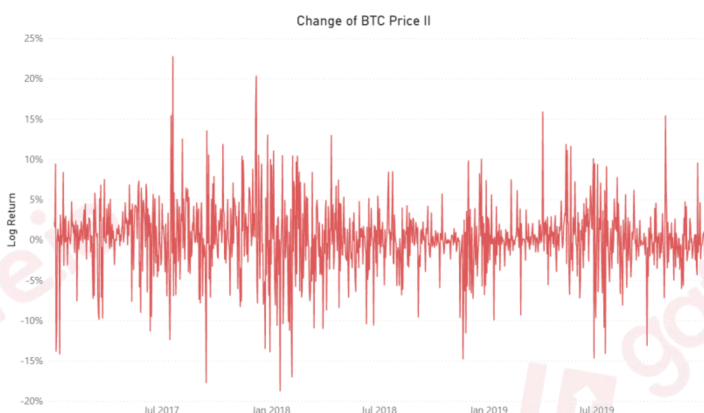Below is the plot of the daily BTC price volatility:

Source: https://cn.investing.com/, *as of May 27, 2020*

As shown in the figure above, the BTC log returns present a sign of clustering and consistency in volatility. In other words, large shocks are often followed by another large shock, while a small fluctuation often follows small fluctuations.

However, this feature was downplayed during the March 12 "Black Swan", where the BTC price saw a drastic slump due to unusual reasons. To present a more reasonable volatility forecast, the abnormal data on March 12 was excluded. Moreover, to avoid an

unnecessary prediction on the BTC volatility for the near future, the historical data sample is extracted only from January 2017 to the end of 2019.

In the first place, we need to carry out an Augmented Dickey–Fuller (ADF)–based unit root test on the AR model. As per the result, t is calculated as –14.22 and p is $1.6×10^{-26}$, suggesting the BTC price volatility is stationary.

Additionally, as the Autocorrelation Function (ACF) is truncated at lag 10 and falls within
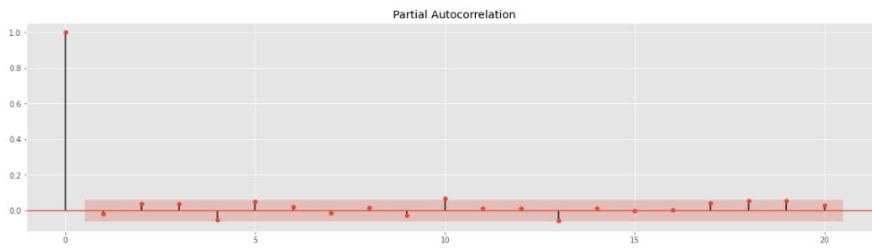
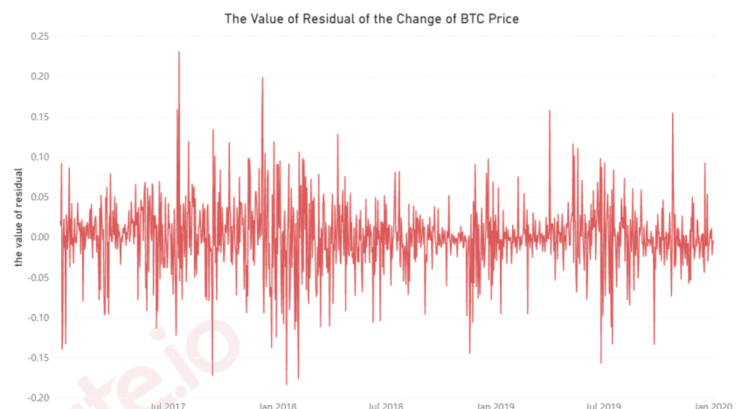the confidence interval (the red region) after lag 10, it indicates an AR (10) model for the variance may be appropriate.

3.2.2 ARCH Test

Before performing the ARCH test on the residuals, an AR (10) model is constructed and the plot of the


residuals of BTC price volatility is made to calculate the value of the residual (t) of the mean equation.

An ACF of the squared residual ε_t^2 suggests that the value of p is between 10^{-19} to 10^{-6}, indicating there is autocorrelation in the residual series. Therefore, it is regarded there exists ARCH effects.

3.2.3 Constructing an ARCH Model

Similar to an AR model, an ACF is needed to determine the order of the ARCH model prior to the construction of an ARCH model.

Partial Autocorrelation
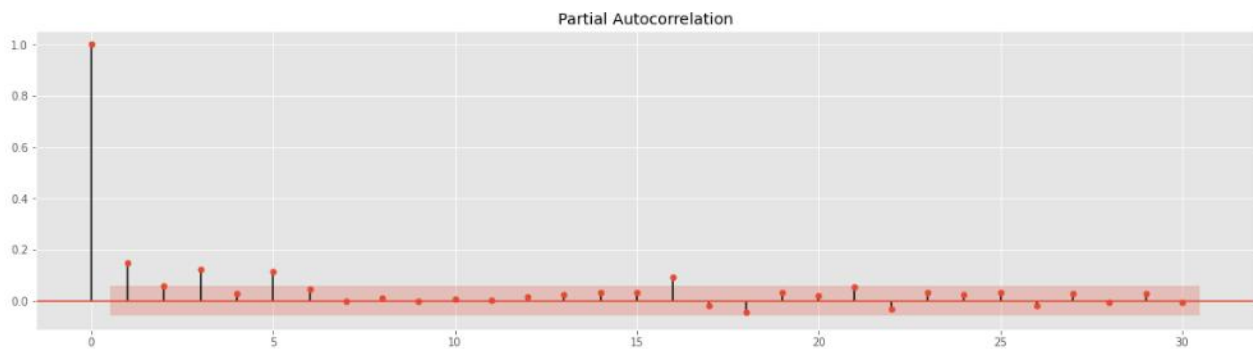
As the above figure shows, the ACF truncates at lag 16 and falls inside the confidence interval after lag 16. In this scenario, an ARCH (16) model might be a good fit.

The ARCH (16) model shows that the value of the R-squared is 0.008, suggesting an insufficient fitting effect. As there are a lot of parameters in the ARCH (16) model, the plot is not displayed here.

3.3 Constructing a GARCH Model

As the ARCH model does not fit the variance, here we also construct a GARCH model. The construction process of a GARCH model is basically the same as that of an ARCH model, without it being necessary to determine the GARCH order in advance, though. Low order GARCH (p, q) models are generally adopted when constructing GARCH models, such as GARCH (1, 1), GARCH (2, 1), and GARCH (1, 2). Below shows some low−order GARCH (p, q) models.

Oder	AIC	BIC
1, 1	−3841.87	−3772.15
1, 2	−3847.67	−3772.97
2, 1	−3833.86	−3759.15
2, 2	−3846.29	−3766.61

As suggested in the figure above, GARCH (2, 1) presents the best effect, where both the Akaike's Information Criteria (AIC) and the Bayesian Information Criteria (BIC) are the lowest. In view of this situation, the GARCH (2, 1) is adopted.

3.3.1 Calculating Parameters

Like the ARCH model, an AR (10) model is applied for the GARCH (2, 1) model, and the construction is conducted with python's ARCH package. Here are the results:

Mean Equation:

$$y_t = 0.001792 - 0.009697a_1 + 0.047321a_2 + 0.026676a_3 - 0.050590a_4 + 0.038114a_5$$

$$+ 0.019494a_6 - 0.001467a_7 + 0.011871a_8 - 0.026352a_9 + 0.066676a_{10}$$

Conditional Heteroskedasticity Equation

$$\sigma_t^2 = 0.000184 + 0.050095\mathfrak{a}_{t-1}^2 + 0.049995\mathfrak{a}_{t-2}^2 + 0.799920\sigma_{t-1}^2$$
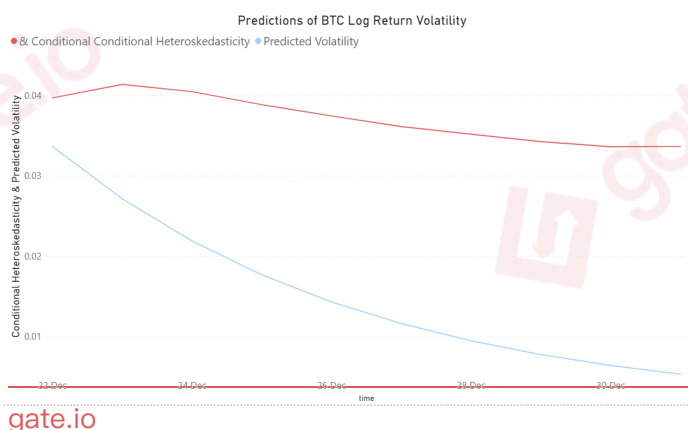
3.3.2 Observing Modeling Effects

As the figure shows, the conditional heteroskedasticity basically captures the BTC log returns. In some sense, it is fair to say that GARCH can be used to predict the BTC price volatility by modeling the BTC log returns.
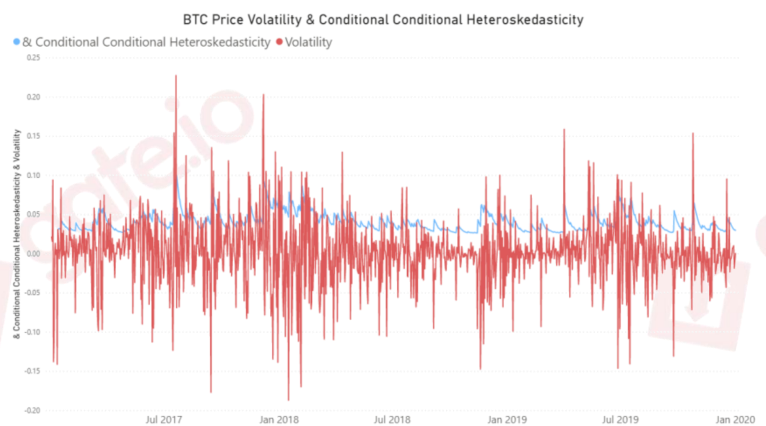
3.3.3 Forecasting BTC Log Return Volatility with GARCH

To better display the modeling effect of the GARCH model for BTC log returns volatility, the training data, collected in the 1,085 days until December 22, 2019,

is used to predict the log return volatility of BTC in the 10 days (December 22, 2019 to December 31, 2019) prior to the modeling.

As observed from the results, while the predicted values are close to the actual values at the first few days, they seem to deviate from the actual situation in the following days. With regard to volatility trend, it is marked by a more significant similarity between the realized values and the predictions, in that the former presents one instance of growth and 8 decreases, while the latter is featured by 9 decreases. It can be concluded that the GARCH model offers more advantages in the forecasting of a volatility trend than in actual volatility.

4 Conclusion

As a new form of financial derivatives, the options contract gives the buyer (the owner or holder of the Option) the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified strike price prior to or on a specified date. It is more user friendly compared to traditional futures contracts. The report highlights the significance of volatility to Options through a review on the category, value, and pricing of Options contracts, leading to the conclusion that **high-volatile Options come with high values.**

The report also illustrates the employment of Delta neutral strategy in Options investments, meanwhile explicitly spelling out that **the returns of Options largely rely on the discrepancy between the IV and the future realized volatility.**

Thanks to their effectiveness in modeling volatilities of various assets, GARCH processes have been extensively used in finance among a lot of professional strategists. Through the construction of a GARCH model, Gate.io Research has found that the GARCH model facilitates **an effective prediction on the volatility trend of BTC log returns, while it presents a weak performance when it comes to the forecast of the volatility itself, as proven by the modeling results in section 3.3.3**. As Gate.io is proceeding with its efforts to launch American options, it remains to be seen whether the GARCH model would have a good effect on predicting the short−term volatility of American options.

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Glossary

[1] Error Term: An error term is a residual variable produced by a statistical or mathematical model, which is created when the model does not fully represent the actual relationship between the independent variables and the dependent variables.

[2] ADF Test: In statistics and econometrics, an ADF test tests the null hypothesis that a unit root is present in a time series sample. The ADF statistic, used in the test, is a negative number. The more negative it is, the stronger the rejection of the hypothesis that there is a unit root at some level of confidence.

Disclaimer

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- Based on due diligence and objective analysis by internal staff, the research analyzes the status quos of digital asset options and should not be treated as the sole basis to predict the development of the digital asset options market.

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